Fast data-derived fundamental spheroidal excitation models with application to UXO identification

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ABSTRACT

Current idealized forward models for electromagnetic induction (EMI) response can be defeated by the characteristic material and geometrical heterogeneity of realistic unexploded ordnance (UXO). A new, physically complete modeling system includes all effects of these heterogeneities and their interactions within the object, in both near and far fields. The model is fast enough for implementation in inversion processing algorithms. A method is demonstrated for deriving the model parameters from straightforward processing of training data from a defined measurement protocol. Depending on the EMI sensor used for measurements, the process of inferring model parameters is more or less ill-posed. More complete data can alleviate the problem. For a given set of training data, special numerical treatment is introduced to take the best advantage of the data and obtain reliable model parameters. This fast model is implemented in a “fingerprint” testing approach in which two different UXOs are identified from the measurement data. Preliminary results showed that this fast model is promising for UXO identification.

Keywords: unexploded ordnance (UXO), electromagnetic induction (EMI), inverse, spheroid, singularity, fingerprint testing, fundamental modes, weighting functions

1 INTRODUCTION

Reliable techniques for subsurface discrimination are urgently needed to reduce the cost of UXO clean ups. As an inverse problem, UXO discrimination requires a fast forward model, i.e. a model calculating EMI responses for prospective targets that may be present. Several fast models have been developed and employed. One of the most successful ones is the dipole model [1,2], in which one approximates a target's response with one or a number of infinitesimal magnetic dipoles, each responding independently to the local value of the transmitted (“primary”) field that strikes it. The dipole model is a good approximation only if the observation position is far enough from the target and if the parts of the object do not interact significantly. However, in UXO detection and discrimination the sensor is often close to the target, and we have shown elsewhere that interaction effects can be very significant [3, 4]. Fast analytical solutions for a spheroid in EMI frequencies range have been developed recently [5,6]. Using these solutions, we have shown elsewhere that some geometrically complex objects can be approximated effectively by a representative spheroid in terms of EMI response [7,8].

For more complicated, materially heterogeneous objects, the dipole or spheroid models are not sufficient, and detailed numerical solution by established methods is too slow. The alternative is a standardized excitation approach, here formulated as a spheroidal mode approach [3,9,10]. We choose a set of fundamental excitation modes in spheroidal coordinates whose linear combination can represent an arbitrary excitation field. The EMI response of the target to each
fundamental mode is obtained and stored. Since the system we are studying is linear, if some general excitation field constitutes a particular superposition (linear combination) of inputs, the response will be a corresponding superposition of outputs. So one just needs to build a library for known UXOs, where the fundamental solutions (i.e. the response of each UXO to fundamental excitation modes) are stored. The EMI response of any UXO to an arbitrary excitation can then be easily constructed from the library data. For the general case, the fundamental solutions can be obtained indirectly from properly designed measurements (training data). A major difficulty in the spheroidal mode approach is that the excitation fields from realistic EMI sensors are usually a combination of the fundamental modes, so the coefficients in the fundamental solution have to be obtained through an inversion procedure, which often suffers from ill-conditioning. We introduced some general techniques to treat the ill-conditioning problem in [10]. In this paper, we further developed those techniques by two additional steps: (1) sort the coefficients of fundamental modes in the primary field and keep only those terms whose coefficients are not negligible; (2) Weighting function are applied so that all eligible observation data has significant contribution to cost function. In the last part we demonstrate the application of this spheroidal mode approach to UXO identification.

2 THE FAST FORWARD MODEL --- EXPRESSING SCATTERED FIELDS WITH FUNDAMENTAL SPHEROIDAL SOLUTIONS

2.1 Decomposing arbitrary primary fields into fundamental modes

As elaborated elsewhere [13,14], in the EMI band we can usually consider the magnetic fields in the air and soil to be irrotational, with the result that they can be expressed as gradients of a scalar potential that satisfies the Laplace equation. Because UXO are typically elongated bodies of revolution, we choose Associated Legendre functions in prolate spheroidal coordinates as the fundamental solution forms. On a spheroid surface at $\xi_0$, some distance away from the sensor, the primary EMI potential field can be written as a summation of fundamental modes:

$$\psi^{pr} = \frac{H_0 d}{2} \sum_{m} \sum_{n=m}^{\infty} \sum_{p=0}^{1} b_{p m n} P_n^m(\eta) P_n^m(\xi_0) T_{p m}(\phi) = \sum_{k=0}^{\infty} b_k \psi_k^{pr}(\eta, \xi_0, \phi)$$

Here and in what follows the coordinate system is centered on the scatterer, $(\eta, \xi, \phi)$ are the standard spheroidal coordinates, $d$ is the chosen inter-focal distance, $P_n^m$ is Associated Legendre functions of the first kind, of order $m$ and degree $n$. $T_{p m}(\phi)$ is $\cos(m\phi)$ for $p = 0$ and is $\sin(m\phi)$ for $p = 1$. The coefficients $b_{p m n}$ can be obtained by using orthogonality relations for the Legendre functions [7]. For the purpose of simplicity, we rewrite the series in the middle of (1) using a condensed subscript notation on the right, with the coefficients series $b_k$ decreasing as $k$ increases. In practice, the series will be truncated, i.e. we will only consider terms $k \leq k_{\text{max}}$. To proceed for an arbitrary object, we define a spheroidal surface $S$ surrounding the object, more or less conforming to its general shape, i.e. we choose some suitable $d$ and $\xi_0$, though the details of that choice are not critical.

In general, the excitation field from a realistic EMI sensor contains mainly a small number of basic modes, with the contribution of other modes (usually high order modes) negligibly small. An example will be shown in Figure 2. In our experience, no more than about the dominant 4 to 6 modes are usually required to describe the field from the GEM-3 sensor. This sensor is described in [15].
2.2 Constructing general responses using fundamental solutions

As for the primary field, the scattered field outside of the target is irrotational and the scattered field potential function satisfies the Laplace equation. For each fundamental unit magnitude spheroidal excitation $\psi_k^p$, we represent the corresponding scattered field with a number of magnetic charges located on (or inside) the auxiliary spheroid surface. The amplitude of the $i^{th}$ magnetic charges $\rho_k^i$ is obtained by the methods described below and then stored before attacking any particular application involving this particular object. When arbitrary excitation fields are decomposed into basic modes (i.e. the $b_k$ in (1) are determined), the scattered field can be easily computed by superposition. One of the essential steps is to obtain the fundamental solutions in the first place, which will be demonstrated in the next section.

The technique is applicable to general 3-D targets [12]. However, most UXOs are bodies of revolution (BOR) or respond like BOR’s. Taking advantage of this property can simplify the problem dramatically, and in this paper we focus on BOR targets. We choose the surrounding spheroid surface so that its symmetry axis is the same as that of the BOR target. The formulation for a BOR has been shown in detail [3,10]. For a basic spheroidal mode excitation $\psi_{pmn}^p = P_{n}^{m}(\eta)P_{m}^{n}(\xi_0)T_{pm}(\phi)$, the scattered field must follow the dependency of $T_{pm}(\phi)$ in $\phi$. Therefore we can construct the fundamental solution with several azimuthal rings of charge, with magnitude $\rho_{pmn}^i T_{pm}$ for the $i^{th}$ ring. The circular rings have the same axis as the BOR and are usually on the surface of the spheroid. The total scattered field at the $j^{th}$ (monostatic) observation point will be

$$H_j = \sum_{pmn} \sum_i b_{pmn}^i \rho_{pmn}^i G_{ipm}^i = \sum_k b_k^i \sum_i H_{ki}^i$$

(2)

where

$$G_{ipm}^i = \int_0^{2\pi} \frac{T_{pm}(\phi')}{4\pi\mu_0 |R_j - R'|^3} (R_j - R') \, r_i \, d\phi'$$

(3)

$$H_{ki}^i = \rho_{pmn}^i G_{ipm}^i$$

(4)

and $r_i$ is the radius of the $i^{th}$ charge ring.

The magnetic charges $\rho_{pmn}^i$ are obtained and stored beforehand. Arbitrary excitation fields are decomposed into basic modes via $b_k^i$, then the scattered field can easily be computed by superposition. One of the essential steps is to obtain the fundamental solutions $\rho_{pmn}^i$, which will be demonstrated in the next section.

3 INFERRING FUNDAMENTAL SOLUTIONS FROM TRAINING DATA

3.1 Algorithm and formulation

The field from realistic sensor is usually a combination of a number of fundamental modes, and we need to infer the response to each single mode from the measurement data. To approach this, we consider below the particular GEM-3 sensor developed by Geophex Ltd.[15], which has distinguished itself in UXO discrimination application.
The GEM-3 we are currently using is mono-static and measures only the component of the scattered field normal to the sensor head, so the forward model is expressed as:

\[ H_j = \sum_{pmn} \sum_i b^j_{pmn} \rho^j_{pmn} g^j_{pmn} = \sum_{k=1}^{k_{pmn}} \sum_i b^j_{ki} H^j_{ki} \]  

with

\[ g^j_{pmn} = \int_0^{2\pi} \frac{T_{pm}(\phi')}{|\mathbf{R}_j - \mathbf{R}'|^3} (\mathbf{R}_j - \mathbf{R}') \cdot \mathbf{N}_j \cdot r_i \cdot d\phi' \]

Where \( j \) is the index of the position \( \mathbf{R}_j \) where that normal field is measured, and \( \mathbf{N}_j \) is the direction normal to sensor head at point \( \mathbf{R}_j \). For given a set of training data (well designed measurements for a specific target), \( \rho^j_{pmn} \) can be determined by minimizing \( F(\rho) = \sum_j |H_j(\rho) - H^d_j|^2 \), where the superscript \( d \) indicates data.

The inverse problem is ill posed for the following reasons: (1) The primary field from GEM-3 sensor contains mainly several dominant fundamental terms (usually low order terms) and very little of the other, usually high order terms. The consequence is that EMI data in this set of training data is not sensitive to the fundamental solution of these high order terms, i.e. \( \partial H / \partial q_k \) is close to zero, so that the Jacobian and Hessian matrix are singular. (2) For a given fundamental excitation, the charges close to the observation points produce a large contribution to the observed scattered field while charges far away don’t have much effect. The unbalanced sensitivity of different points can cause numerical instability and is another source of ill-conditioning.

Two approaches were discussed in [12] -- simple truncation and the LM approach. In the truncation approach, the non-significant terms are truncated so that the ill-conditioning caused by the first factor is reduced. The truncated system is still ill-conditioned because of the second factor. The LM approach deals with general ill-conditioning problems. Altogether, in [12] we truncated the high order terms and also treated the ill-conditioning with the LM algorithm. Since we kept some low order terms which were in fact non-significant, the system was highly ill-conditioned and required a long time to converge.

In this paper we will combine the two approaches and develop a fast and stable procedure. Instead of truncating high order term per se, we sort the summation of \( b_k \) for a given set of training data and keep only the dominant ones for the case at hand. The fundamental solutions of these dominant modes are then inferred from training data via the LM approach. This approach truncates all the non-essential modes, alleviating the ill-conditioning caused by factor one, reducing the number of unknowns and thus speeding up the inversion processing dramatically.

Usually the training data contains measurements for a target at different distances and orientations relative to sensor. The magnitude of measurement data drops rapidly as the target moves away from the sensor. The result is that the data measured at greater distance will be overshadowed by those measured at near field and will not contribute much to the objective function. A weighting function is needed so that both near and far field data are taken into account in a balanced way.

The objective function with weighting function \( w_j \) is

\[ F(\rho) = \sum_j w^2_j \left| H_j(\rho) - H^d_j \right|^2 = \sum_j w^2_j \left| \sum_k b^j_k H^j_{ki} - H^d_j \right|^2 \]  

The iterative scheme according to LM algorithm will be
\[
\rho_{e+1} = \rho_{e} - \left[ J_w^T J_w + \mu I \right]^{-1} \cdot \left( J_w^T H_w \right)
\]  \hspace{1cm} (7)

Where \((J_w)_{jk} = w_j \frac{\partial H_j}{\partial \rho_k}\), \(H_{wj} = w_j \left[ H_j (\rho) - H_j^d \right] \)

The term \(\mu I\) is for regularization, and \(\rho_{e}\) is a vector of \(\rho_{k}^e\) evaluated at the \(e^{th}\) iteration.

The weighting functions are chosen based on the following considerations: (1) Both near and far field data should make sufficient contributions; (2) the far field is more vulnerable to noise, so the near field should be given more weight; (3) data below the noise level should be discounted; and (4) because terms deemed less significant were truncated as the inversion system was constructed, if the primary field at a particular observation point contains significant contributions from the truncated terms, the data for this point should be deleted. Based on these criteria, the weighting functions are set as:

\[
w_j = \begin{cases} 
0 & \text{if } \left| H_j^d \right| \leq \epsilon_{\text{bkg}} \\
0 & \text{if } \left| \sum_{p} \sum_{m} \sum_{n} \left| b_{pmn}^j \right| - \left| b_{000}^j \right| \right| / \sum_{k} \left| b_k^j \right| \geq \epsilon_{\text{ho}} \\
\frac{1}{\left( \sum_{k=1}^{k_{\text{max}}} \left| b_k^j \right| + \epsilon_{\text{floor}} \right)} & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (8)

The term \(\sum_{k=1}^{k_{\text{max}}} \left| b_k^j \right|\) is somewhat similar to but milder than the sensitivity of charges to \(j^{th}\) observation. Unlike weighting with sensitivity as practiced by some authors, the weighting function here ensures that the near field data makes more of a contribution than far field data. The floor term \(\epsilon_{\text{floor}}\) is added to avoid numerical difficulty when \(\sum_{k=1}^{k_{\text{max}}} \left| b_k^j \right|\) is close to zero. The data are discounted when the magnitude is too small (\(\left| H_j^d \right| \leq \epsilon_{\text{bkg}}\)) or when the higher order terms produce a large fraction of the input, as determined relative to \(\epsilon_{\text{ho}}\).

3.2 Example and results

To demonstrate the approach for inferring fundamental solutions, we choose a UXO (Figure 1) as an example. A 1x3 surrounding prolate spheroid is used for basic mode decomposition. The measurement setup is shown on the right in Figure 1. The GEM3 sensor is fixed right under the center of a wooden grid, and the UXO (with different orientation --- nose up, nose down and horizontal) is moved along the two lines crossing at the center of the grid ----west to east and north to south, with step size 10 cm and 13 points each line. The wooden grid can be moved up and down to vary the distance between the target and sensor.
Surrounding Spheroid

Figure 1. Left: UXO 28 cm in length and about 8.3 cm in diameter at its widest point. The surrounding prolate spheroid surface has dimensions $2a = 12$ cm, $2b = 36$ cm, with the same center and symmetrical axis as the UXO. Right: measurement setup for the training data, GEM sensor is fixed under the wooden grid and the target moves along top of the grid.

We choose two set of measurements (distance of sensor to measurement surface 21.5cm and 36.5cm) as the training data, the summation of the coefficients of each term (i.e. $\sum_j |b_{j|pmn}|$) are plotted in Figure 2, sorted (renumbered) according to magnitude on the right. We see that there are only 4 dominant terms: pmn=001, 011, 111 and 002. The first three terms correspond to uniform magnetic field at three orthogonal directions. For each dominant mode, we express the scattered field with $n_{source} = 6$ rings of charges, distributed uniformly along the axis of the spheroid. The magnitudes of the charge rings for solutions corresponding to each spheroidal excitation mode $\Psi^s_k$ are inferred from training data and saved in the library.

![Figure 2](image)

Figure 2. Left: Summation of $|b_{j|pmn}|$ over all observation points as the sensor moves over a two lines across the center of the grid, from west to east and from north to source. Right: Similar to the plot on the left, but sorted in descending order.

To check the accuracy of the inferred fundamental solutions, we measured the scattered field at locations different from that used to produce the training data, and compared results with those calculated using the fundamental solutions. The
test data are from sensor distances of 26.5cm and 46.5cm, with the target traveling from west to east. Figure 3 shows the comparison of results for 90Hz, for both 26.5cm and 46.5cm distances, real and imaginary part for each case. The left figures are real part, with UXO nose up, nose down and horizontal, 13 points for each orientation. The right figures are for imaginary part of the same three orientations. The modeled data agrees well with measurement data, even for the 46.5cm case where signal to noise level is rather low.

Increasing the number of fundamental modes used within a reasonable range (i.e. less than 10 in this case) will improve accuracy a little bit but will increase the computational cost substantially. Further, it is not always true that accuracy is measurably increased, because the data may contain very weak information on the added modes (Figure 4). Adding those terms into unknowns will cause exacerbate ill-conditioning problems, so that accurate fundamental solutions are difficult to obtain, even for the dominant modes.

![Figure 3. EMI response (90Hz) constructed from 4 fundamental solutions (green triangles) compared to measurements (squares).](image)
Figure 4. EMI response (90Hz) at 46.5cm height constructed from 8 fundamental solutions, showing a better match with measured data than for 4 fundamental modes, especially for horizontal position.

4 APPLICATION OF FUNDAMENTAL MODEL APPROACH IN UXO IDENTIFICATION

The most straightforward way to approach UXO identification is "fingerprinting" i.e. scattered field pattern matching [17]. If there is only a small number of UXO candidates in a specific field, we can obtain the signature (the fundamental solutions here) of each candidate prior to surveying. Then one simply tests each candidate against measured data by optimizing possible positions and orientations, and classifies the anomalies according to the goodness of fit.

As an example, consider two UXO items designated as UXO#5 and UXO#6 (Figure 5). Their fundamental solutions for mode pmn=001,011,111 and 002 are inferred from training data for each of them in isolation.

Figure 5. Two UXO models studied for fingerprint matching approach
For the test data, the two targets were placed in foam molds on a square board (the "measurement surface") 24.2cm above the sensor (we used 64cm diameter GEM-3 for this set of measurement, instead of the 40 cm GEM-3 in the data above). The foam mold was moved over a 6x6 grid on the surface and the scattered fields were recorded. The targets are separated (~ 40 cm) so that we can neglect their interactions in the model, although their signals overlap spatially.

To identify the two anomalies from the test data, we build up an inverse problem to minimize

$$F(\rho) = \sum_j |H_j^{#5} + H_j^{#6} - H_j^d|^2$$

where model data $H_j^{#5} + H_j^{#6}$ are calculated from fundamental solutions of UXO#5 and UXO#6, their locations and orientations are free parameters to be inferred in the inversion process. At optimal locations and orientations, the modeled data match virtually perfectly with the measurement data (Figure 6), which indicated that the two anomalies are indeed UXO#5 and UXO#6 (otherwise it is unlikely that we can obtain such a good match).

![Figure 6. Scattered field (real part, 90Hz) over a 6x6 grid when two targets are presented together, showing a comparison of measured with modeled data for the optimal locations and orientations.](image)

**5 SUMMARY DISCUSSION**

A fundamental mode approach was developed for fast calculation of the complete EMI response of complicated 3-D objects. For prospective targets, the fundamental solutions are obtained and stored in a library. The response of these targets to arbitrary EMI excitation can then be constructed quickly by simple superposition of the fundamental solutions. A robust and fast inversion procedure is developed to infer fundamental solutions from training data measured by GEM-3 sensor. The modes that don’t contribute much to the measured data are cut off in the inversion procedure to reduce ill-conditioning and computational burden. Weighting functions are also employed in the optimization to balance data obtained at different distances, to reduce noise effects and suppress ill-conditioning. A Levenberg-Marquardt approach is finally employed in the inversion calculations where some degree of ill-conditioning difficulty still remains. An example with a model UXO showed that the procedure quickly gives the fundamental solutions for dominant excitation modes. These solutions are used to calculate quick and accurate responses to the GEM-3 sensor within a reasonable observation region. The forward model based on fundamental solutions is then applied to UXO identification via a pattern matching approach in which two UXO’s are present simultaneously with their signals overlapping in space. Optimizing for target
locations and orientations produces a good match between measured and modeled data, verifying the presence of the two UXO's.

The sensors we studied produced only a few dominant excitation modes. This simplified the inverse problem for inferring fundamental solutions. At the same time, it also means that it is hard to obtain accurate higher order information from measurements with only this single sensor, as might be necessary for UXO discrimination in other cases. For more complicated objects, we may also need better ways of distributing the magnetic charges in the model to obtain greater accuracy without excessive computation cost. A better design for training data acquisition (especially bistatic data or data for three components of $\mathbf{H}$) may also help to reduce ill-conditioning problems.

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