ELECTRICAL IMPEDANCE TOMOGRAPHY FOR GEOPHYSICAL APPLICATIONS

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Abstract

We report on the development of a new software package for image reconstruction in three-dimensional Electrical Impedance Tomography (EIT). Designed for large-scale computations, the software includes mesh generation utilities, a simulator of the quasi-static electrical potential fields based on finite elements and regularized inversion algorithms. We describe in brief some of its main functions and characteristics to demonstrate its utility and efficiency in handling high dimensional geophysical models.

Introduction

We present a software framework for the electrical impedance tomography problem in three axial dimensions. In this overview we consider the forward problem for the underlying electromagnetic model, the sensitivity of the measurements to the electrical properties of the medium of interest, the computation of the nonlinear solution using regularization methods and the visualization of the results. The motivation for developing this software is to exploit the recent advances in inverse problem theory and state of the art models for surface and borehole electrodes while implementing computationally efficient algorithms aimed to reduce the complexity introduced by the dimensionality of the resulting models. Incorporating meshing and visualization utilities this Matlab [4] based software is suitable for processing data from any EIT or resistivity survey acquisition instrument on a personal computer. The software relates to the releases of RES3DINV [3], R3 by Binley et al. [1], and EIDORS 3D by Polydorides et al. [6], but it differs in various ways as it is designed to handle models with realistic large-scale complexity.

Forward Modeling and Simulation

We consider the complete electrode model for 3D electrical impedance tomography, although simpler electrode models are also implemented, see for example the review [5]. The model was introduced and analyzed by Somersalo et al. [8] and subsequently implemented via numerical approximation by Vauhkonen et al. [11], Borsic [2] and Polydorides et al. [6]. Assume a domain $\Omega$ with boundary $\partial \Omega$, and electrical admittivity $\gamma = \sigma + i \omega \epsilon$, where $\sigma$ is the electrical conductivity, $\epsilon$ the dielectric permittivity, $\omega$ the angular frequency of the applied excitation and $i = \sqrt{-1}$. Electrically isotropic or anisotropic domains can be considered, in the later case the electrical properties admit tensor definitions (with respect to the Cartesian frame), but here we address the simple isotropic case with $|\gamma| > 0$. When a number $L$ of borehole and surface electrodes are attached at $\partial \Omega_E \subset \partial \Omega$ and excited in a pair drive mode, the resulting electric potential field $u$ satisfies

$$\nabla \cdot \gamma \nabla u = 0 \quad \text{in} \ \Omega$$

(1)

For the two excited electrodes, the impressed currents of magnitude $I_l$ satisfy the Neumann conditions

$$\int ds \ \gamma \nabla u \cdot \hat{n} = I_l$$

(2)

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where the integral is taken on the surface of the electrode and \( \hat{n} \) the outward unit normal there. For the remaining electrodes we impose

\[
\gamma \nabla u \cdot \hat{n} = 0
\]

The voltage measurement recorded by the \( k \)th electrode with contact impedance \( z_k \) is given by

\[
V_k = u + z_k \gamma \nabla u \cdot \hat{n}
\]

At the model termination boundaries \( t_1, t_2, \ldots \) we assume absorbing boundary conditions

\[
u + z_{t_i} \gamma \nabla u \cdot \hat{n} = 0
\]

Enforcing the required charge conservation principle \( \sum_{k=1}^{L} l_k = 0 \), the model admits a unique solution when a choice of ground is made, i.e. for a point \( g \in \Omega \), we enforce \( u(g) = 0 \). The finite element approximation of this model relies on discretizing the domain of interest in linear tetrahedral elements. Compatible with various mesh generators, this package includes its own high level meshing utilities aimed to ease the manipulation of the boundary for electrode assignment and borehole allocation. Moreover, the mesh’s outer surfaces and electrode-domain interfaces are automatically labeled for efficient application of boundary and mesh termination conditions. For a given tessellation the domain’s electrical properties, are defined using constant or linear basis functions having local support on elements.

In the adopted finite element method the forward model is approximated by a system of linear equations involving a sparse, symmetric and positive definite (SSPD) coefficients matrix. Under some mild continuity assumptions on the electrical properties, the matrix is well-conditioned, however its dimension scales to that of the nodes (vertices) in the finite element model. As such, the computation of the forward solution may become computationally expensive or even prohibitive for moderate architectures and algorithms relying on inverting the coefficients matrix. To overcome this limitation, the software incorporates an iterative solver customized for multithread processing of SSPD systems called Pardiso [7]. To provide an indication of the computational benefits we present in table 1 some timing information from a benchmark test on a model with about 98000 nodes and 540000 elements. The results are based on an 8-core, shared memory PC architecture based on two quad core Xeon 5355 CPUs, with an internal clock of 2.66 GHz and a front side bus speed of 1.33 GHz, operating Ubuntu Linux 7.05 64-bit, running Matlab 2009a 64-bit.

<table>
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<th>2 threads</th>
<th>4 threads</th>
<th>8 threads</th>
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<td>34.21</td>
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<td>19.56</td>
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</tbody>
</table>

Table 1. CPU times in seconds for 30 forward solutions corresponding to different current patterns. Note the substantial improvement in speed provided by the Pardiso solver.

Moreover, to treat the situations where the forward problem has to be solved for a large number of current patterns, i.e. greater than \( L \); the number of the system’s electrodes, we propose the use of canonical patterns introduced by Borsic in [2], and this bounds the number of right hand side vectors to a maximum of \( L \). In the conventional four electrode data acquisition protocol which implements pair drive current patterns, one typically considers right hand side vectors for the fem problem that have zero entries apart from the two excited electrodes which are typically assigned values \(+I_l\) and
In order to reduce the number of forward problems that need be solved, we remark that all possible forward solutions can be synthesized by linear operations from a total of $L$ solutions that correspond to the canonic patterns defined by the vectors \{1−1/L,1−1/L,...,−1/L\} ∈ $\mathbb{R}^L$, \{−1/L,1−1/L,...,−1/L\} ∈ $\mathbb{R}^L$ and so on. To exploit further the recent advances in multithread computing, a future release of this software will facilitate GPU processing, thereby maximizing the utilization of the available resources on a single machine.

As it is well known, forward solutions are also needed in the computation of the sensitivity of the system, a linear operator mapping perturbations in the electrical properties in the interior of the domain to the boundary; surface of borehole, potential measurements. Details on the derivation and computation of this $m$ by $n$ matrix, where $m$ the number of boundary data and $n$ the dimension of the electrical parameter space can be found in [6]. As it well known the computation of the linearized sensitivity map (matrix) requires the forward solutions for all direct and adjoint current patterns. In three-dimensional models, $n$ can be very large; whereas $m$ may be smaller by comparison, and this may introduce limitations in regards to the storage and thereafter processing of this matrix in the inverse solver. To alleviate this difficulty, we propose an automatic change of basis for the electrical properties, where the new basis, used exclusively for the inverse fitting problem, has a lower dimension. This change is achieved by projecting the elements of the sensitivity into regions of interest according to their topology in the mesh. This yields a projected sensitivity matrix of manageable dimensions, which we subsequently use to formulate the (linearized) inverse problem.

### Inverse Problem

The nonlinear inverse problem is addressed in the context of Newton-Raphson iterative algorithm with Tikhonov type regularization. Beginning with an initial feasible estimate on the solution, our software implements a “linearization – regularization” strategy where the sensitivity matrix is dynamically updated and the next solution estimate is obtained using a linear least squares fitting and some a priori information that is available on the solution. The generalized form of the inverse problem considered is given by

$$\gamma^* = \operatorname{argmin}\left\{||F(\gamma)−d||_{\Sigma^{-1}}^2 + ||D(\gamma−\gamma_0)||_{\Sigma^{-1}}^2\right\}$$

(6)

where $F(\gamma)$ the data corresponding to a forward model with admittivity $\gamma$, $d$ is the measurements, $D$ is the regularization matrix that represents the prior information assumptions, $\gamma_0$ is a initial guess on the solution and $\Sigma_{d}$, $\Sigma_{\gamma}$ are the noise and prior information uncertainty covariance matrices respectively. To ensure convergence, a line search algorithm is used to optimize the size of the regularized step solution. Alternative regularization options include smoothness or sparsity imposing priors, where the norms in the minimization become $\ell_2$ or $\ell_1$ and total variation respectively. The nonlinear regression problem (6) is solved iteratively, after linearizing the argument at feasible estimates $\gamma_t$, where $F$ is approximated by its first-order Taylor expansion at these points. Effectively this requires the computation of the Jacobian matrix $J = \nabla_x F$ whose elements indicate how the measurements will change for small perturbations in the admittivity of the elements. In a high-dimensional model the computation of this matrix becomes cumbersome in terms of storage and speed. Moreover, forming and inverting the normal equations coefficient matrix $J^T J$, (prime denotes transposition), required in solving the linearized problem becomes prohibitive even in cases with heavily underdetermined models. To cope with this bottleneck, the code implements an element grouping-weighting strategy that yields a projection operator $P$ that maps the elements of the fine grid to a new coarser mesh of manageable dimension. Effectively, the data term $J^T d$ encountered in the calculations (of dimension $m$) is approximated by $J^T \tilde{\gamma} \approx J P \tilde{\gamma} = \tilde{J} \tilde{\gamma}$, where the tilded quantities are defined on the coarse grid.
Visualization of Results

Reconstructed images of the conductivity and or permittivity of the domain are exported along with the inverse problem mesh in vtk format. This allows their three-dimensional manipulation for diagnosis and visualization in a timely and memory efficient manner. The exported files are readily available to be imported in various visualization packages such as Paraview [9] and Mayavi [10], which allow the application of various graphical filters such as extraction of planar solutions, isosurfaces, gradients, etc. Apart from the electrical parameter the software is suitable for visualizing the induced electrical potential and current density fields.

![Visualization of borehole electrode arrangements and admittance models with Paraview.](image)

Conclusions

In this paper we have presented the main features of a new software for three-dimensional Electrical Impedance Tomography. We have outlined its main utility functions in solving the forward problem, and gave indicative figures of its computational performance in handling high-dimensional models. The code implements a nonlinear inverse solver based on a Newton-Raphson algorithm with a Tikhonov regularization formulating at the linearized step solutions. Some simulated results from borehole electrical resistivity tomography are also presented.
Figure 2: Visualization of borehole electrical potential and current density fields with Paraview.

References